Traditional and New MIP Models for Production Scheduling With In-Situ Grade Variability

SALIH RAMAZAN1 AND ROUSSOS DIMITRAKOPOULOS2

ABSTRACT

Mixed integer programming (MIP) has become a common approach for optimizing production schedules of open pit mines since the 1960s. However, MIP has been found to be limited by: (a) feasibility in generating optimal solutions with practical mining schedules; and (b) inability to deal with in-situ variability of orebodies. In looking into these shortcomings, this paper presents a general production scheduling method for multi-element deposits in open pit mines. The method is subsequently used to optimize the production schedule in a nickel laterite deposit. The application confirms the weaknesses of MIP formulations mentioned above. An alternative MIP formulation is then presented and applied to the same deposit. The results of the new formulations show that the new MIP model can overcome the above shortcomings and generate practical mining schedules with a higher chance of achieving planned production targets than traditional MIP schedules.

Keywords: Mixed integer programming, mine production scheduling, grade uncertainty.

1. INTRODUCTION

Optimization of the long-term production scheduling mechanism is a major step in mine planning. It aims to maximize the overall discounted net revenue to be generated from a mining project within the operational constraints, such as mining slope, grade blending, ore production and mining capacity. Mathematical modeling methods such as mixed integer programming (MIP) and linear programming (LP) are common tools in optimizing mine production scheduling. There have been several efforts in developing suitable mathematical models. Johnson [1] introduced LP to optimizing mine planning, but it has been stated that LP models have the problem of partial mining of blocks [2]. Gershon [3] discussed an LP approach together with MIP

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models for optimizing mine scheduling. Notable also is the work by Dagdelen, who formulated long-term production scheduling as an MIP mathematical model, and applied the Lagrangian and sub-gradient methods to solve the formulations [4].

A key limit in past MIP models has been difficulty in solving large problems, as these require a substantial number of binary variables. The attempted LP-based models often generate fractional mining of blocks, leading to the design becoming infeasible and/or non-optimal. Ramazan [5] proposes a new method based on the fundamental tree concept, and substantially decreases the number of binary variables in MIP formulations for long-term production scheduling. Ramazan and Dimitrakopoulos [6] present alternative methodologies useful in reducing the number of binary variables in MIP models. Although the above methods significantly decrease the number of binary variables required and enhance the application of MIP in large mineral deposits, in-situ orebody variability is not considered and all inputs are considered without uncertainty. Due to size issues, MIP applications in complex multi-element deposits, such as nickel laterites or iron ore, are very limited [7].

The dramatic conceptual and economic differences in optimizing open pit mine design for risk-based orebody frameworks are documented in the technical literature [8]. The need to consider risk as well as in-situ variability in production scheduling has also been shown in the past [9]. Similarly, others show the risk quantification in traditional short-term production scheduling, using MIP for a limited number of binary variables [10]. Additional studies show how traditional optimization models generate infeasible schedules in terms of production requirements [7]. Dimitrakopoulos and Ramazan suggest a probabilistic approach to optimizing production schedules of complex multi-elements deposits [11], minimizing the risk of deviations from production targets.

This paper first develops an MIP formulation for optimizing long-term production scheduling in stratiform, multi-element deposits, then applies the formulation to a Ni-Co laterite deposit. Issues of infeasibility associated with the traditional MIP models and their limitations with respect to practical scheduling are investigated. An alternative MIP formulation is then developed. This new formulation considers the probability of blocks being scheduled in a given production period, thereby dealing with in-situ grade variability and practical issues in scheduling patterns.

2. A TRADITIONAL MIP FORMULATION FOR PRODUCTION SCHEDULING

In long-term production scheduling for open pit mines, MIP models are generally used to maximize the overall discounted economic value, or net present value (NPV), of a mining project. The general MIP form of such production scheduling can be represented as follows.
TRADITIONAL AND NEW MIP MODELS

The objective function:

Maximize \( \sum_{t=1}^{T} \sum_{n=1}^{N} c_{n}^{t} * x_{n}^{t} \)  \hspace{1cm} (1)

where \( T \) is the maximum number of scheduling periods, \( N \) is the total number of blocks to be scheduled, \( c_{n}^{t} \) is the NPV to be generated by mining block \( n \) in period \( t \) and \( x_{n}^{t} \) is a binary variable, equal to 1 if the block \( n \) is to be mined in period \( t \), otherwise 0.

Grade blending constraints:

(a) Upper bound constraints: The average grade of the material sent to the mill has to be less than or equal to a certain grade value, \( G_{\text{max}} \), for each period, \( t \)

\[
\sum_{n=1}^{N} (g_{n} - G_{\text{max}}) * O_{n} * x_{n}^{t} \leq 0 \hspace{1cm} (2)
\]

where \( g_{n} \) is the average grade of block \( n \), and \( O_{n} \) is the ore tonnage in block \( n \).

(b) Lower bound constraints: The average grade of the material sent to the mill has to be greater than or equal to a certain value, \( G_{\text{min}} \), for each period, \( t \)

\[
\sum_{n=1}^{N} (g_{n} - G_{\text{min}}) * O_{n} * x_{n}^{t} \geq 0 \hspace{1cm} (3)
\]

Reserve constraints:

Reserve constraints are constructed for each of the blocks to state that all the blocks in the model considered have to be mined once

\[
\sum_{t=1}^{T} x_{n}^{t} = 1 \hspace{1cm} (4)
\]

Generally, the orebody model contains many blocks and it is very difficult, or impossible, to generate a solution through MIP formulations if they are applied to the whole orebody model. Therefore, it is often necessary to consider only applying the formulations to the blocks within the ultimate pit limits and requiring all the blocks to be mined in one of the periods.

Processing capacity constraints:

(a) Upper bound: The total tons of ore processed cannot be more than the processing capacity (\( PC_{\text{max}} \)) in any period, \( t \)

\[
\sum_{n=1}^{N} (O_{n} * x_{n}^{t}) \leq PC_{\text{max}} \hspace{1cm} (5)
\]
(b) Lower bound: The total tons of ore processed cannot be less than a certain amount ($PC_{\min}$) each period, $t$

\[ \sum_{n=1}^{N} (O_n + x_n^t) \geq PC_{\min} \quad \text{(6)} \]

*Mining capacity:*

The total amount of material (waste and ore) to be mined cannot be more than the total available equipment capacity ($MC_{\max}$) for each period, $t$

\[ \sum_{n=1}^{N} (O_n + W_n) * x_n^t \leq MC_{\max} \quad \text{(7)} \]

where $W_n$ is the tonnage of waste material within block $n$.

A lower bound may need to be implemented if it is important for the MIP model to produce balanced waste production throughout the periods, as follows

\[ \sum_{n=1}^{N} (O_n + W_n) * x_n^t \geq MC_{\min} \quad \text{(8)} \]

*Slope constraints:*

All the overlying blocks that must be mined before mining a given block have to be determined. This can be implemented through one or more cone templates representing the required wall slopes of the open pit mine. There are two ways of implementing these constraints:

(a) Using one constraint for each block per period:

\[ Yx_k^t - \sum_{r=1}^{t} x_r^t \leq 0, \quad t = 1, 2, 3, \ldots, T \quad \text{(9a)} \]

where, $k$ is the index of a block considered for excavation in period $t$, $Y$ is the total number of blocks overlying block $k$ and $y$ is the index for $Y$ overlying blocks.

(b) Using Y-constraints for each block per period:

\[ x_k^t - \sum_{r=1}^{t} x_r^t \leq 0, \quad t = 1, 2, 3, \ldots, T \quad \text{(9b)} \]

It should be noted that the MIP formulations presented do not consider the smoothness of the scheduled patterns, which relates to equipment movement in a period and to equipment access. Geological uncertainty is also ignored in this traditional optimization model.
3. APPLICATION OF TRADITIONAL MIP IN A NICKEL LATERITE DEPOSIT

The mine's estimated deposit model includes nickel, cobalt, magnesium and aluminum grades, volume of percent rock, and thickness of two different geological layers. The main mineral considered for profit in this project is nickel, whilst cobalt provides a small contribution to overall mine profit. Too much or too little magnesium or aluminum may increase the processing cost significantly, since these elements affect acid consumption. The orebody model used contains 2030 (58 x 35) blocks with 40 m x 40 m dimensions along north-south and east-west directions, with variable thickness depending on the availability of ore. Each orebody model block may contain both ore and waste material. The deposit is estimated to contain around 28.9 million tons of ore, with an average grade of about 1.30% Ni, 4.50% Mg, 0.58% Al, and a total of approximately 47.4 million tons of material. A production schedule for the deposit is developed to maximize the total NPV at an 8% discount rate, meeting the production targets in terms of periodical ore production, with certain grades of Ni, Al and Mg for a 3-year mine life. The binding constraints implemented in the optimization process are provided in Table 1. Ni grade and ore tonnage distributions for the estimated model are shown in Figure 1.

The estimated deposit model is used as input for the traditional MIP scheduling model, which is applied to maximize the total NPV of the project. Figure 2a shows the plan view of the schedule produced by the MIP model and Table 2 summarizes the results, showing that the MIP model constraints are respected. However, since this MIP model does not consider the practicality of the scheduling patterns, the resultant schedule is widely spread in individual periods and of questionable feasibility. For example, equipment movement in a single period is unrealistic and blocks may be difficult to excavate in the absence of space to accommodate equipment access. As the scheduling pattern obtained from the MIP model is impractical, the original scheduling images are smoothed according to their location and scheduled periods. Figure 2b shows the image after smoothing and Table 3 summarizes results from the smoothed schedule. Although the feasibility of the scheduling pattern in terms of required equipment mobility in a single period is improved over the original MIP

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<tr>
<td>Mg (%)</td>
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<td>5.0</td>
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Fig. 1. Nickel grade (left) and ore tonnage (right) distributions of the estimated model.

Fig. 2. Plan view of the schedule obtained by using the kriging model: (a) is the output of MIP model; (b) is the smoothed image of (a).

output, it is still questionable. In addition to the implementation issues, Table 3 shows that ore production constraints are violated in the first and second years by smoothing the schedule, and Al content (%) constraints in the second year.
Table 2. Summary results from the MIP schedule of the estimated model. T. tons is the total tonnage; O. tons is the ore tonnage; UEV is the undiscounted economic value; S./Ave is the sum of the columns for tonnage and economic values, and the average of the columns representing grades.

<table>
<thead>
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<th>Time (years)</th>
<th>T. tons ($10^6$)</th>
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Table 3. Summary results after smoothing the MIP schedule of the estimated model. T. tons is the total tonnage; O. tons is the ore tonnage; UEV is the undiscounted economic value; S./Ave is the sum of the columns for tonnage and economic values, and the average of the columns representing grades.

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4. SCHEDULES OF THE SIMULATED OREBODY MODELS AND THE ANALYSIS

Schedules are produced for 30 jointly simulated models [12, 13] of the nickel laterite deposit, using the same MIP model as that used for the estimated model schedule in the previous section. As before, the deposit models include Ni, Co, Mg, Al, volume of percent rock and thickness of two layers. Figure 3 shows the Ni grade distribution of 5 simulated orebody models and Figure 4 shows the ore tonnage distributions which, although similar in general, have significant local differences. Figure 5 presents the scheduling patterns of these 5 simulated orebody models as direct output of the MIP model (left) and after smoothing the image (right).

As shown in Figure 5, there are significant differences among the scheduling patterns. These patterns are also different from the pattern of the traditional schedule shown in Figure 2. Differences in the scheduling patterns are caused by in-situ geological variability, which is the main reason for failures to meet planned production targets in mine projects. Furthermore, the schedules obtained directly from the MIP model are not practical. Smoothing the patterns, by considering the
location of the blocks and scheduling periods of the surrounding blocks to improve their practicality, results in substantial differences between the smoothed images and the direct MIP output. Changing the scheduling periods of many blocks often causes the schedule to be infeasible for the operational constraints set for ore tons and element grades. And, if the schedule is still feasible, it is probably far from being optimal. Clearly, the application of traditional optimization formulations for production scheduling is weak in the presence of in-situ variability and uncertainty. At the same time, the ability to simulate equally probable realizations of an orebody is not sufficient if the objective is to generate a schedule that can minimize deviations of forecasts from production, let alone an ‘optimal’ production schedule.

5. AN ALTERNATIVE MIP MODEL FORMULATION

Using the schedules of the simulated orebody models derived with the formulation in Section 2, the probability of each block being excavated in a given period can be
readily calculated. A zero probability for a given period means a block will not be excavated in that period. A probability of one means a block will be excavated in that period. The blocks that have probabilities between zero and one of being scheduled in a period are considered in the new optimization model. The objective function of an alternative MIP model is constructed as

$$\text{Maximize} \quad \sum_{i=1}^{T} \left[ \sum_{n=1}^{N} c_n^i \times x_n^i - \sum_{m}^{M} w \times d_m^i \right]$$

where, $T$ is the maximum number of scheduling periods; $N$ is the total number of blocks to be scheduled; $v_n^i$ is the NPV to be generated by mining block $n$ in period $i$; $p_n^i$ is the probability of block $i$ to be scheduled in period $i$; $c_n^i$ is $(v_n^i \times p_n^i)$, $x_n^i$ is a binary variable, equal to 1 if the block $i$ is to be mined in period $i$, 0 otherwise; $w$ is the cost of a unit deviation associated with generating a smooth scheduling pattern; $d_m^i$ is the deviation from the smooth pattern when mining block $m$; $M$ is the total number of blocks that smoothness constraints are applied.
Fig. 5. Schedules obtained using the traditional MIP optimization model for 5 simulated orebody model inputs. For each model, the figure on the left shows the direct output of the MIP model, while the figure on the right shows the schedule after smoothing.

The first part of the objective function deals with maximizing the probability of the blocks being scheduled in the period predicted by the simulated orebody models. It should be noted that the probabilities are obtained from the schedules that are generated using MIP models that aim to maximize NPV. Therefore, the objective function already considers scheduling the higher economic value blocks in the earlier periods. However, a criterion could be included in the new MIP model to decide which blocks would be scheduled in which period, when the blocks have equal probabilities. For example, say Block 1 has an economic value of $100, and Block 2 $80. If both of the blocks have 60% probability of being mined in the first year and 40% in the second year, and only one block could be mined in the first year due to the operational constraints, it would be preferable to mine Block 1 in the first year. This logic is represented in the objective function when multiplying probabilities with NPV.
The second part of the objective function deals with the practical feasibility of the scheduling patterns. In particular, equipment must be able to access blocks to be mined in a given period, and the movement of large mining equipment needs to be minimized. Formulations for smooth mining are detailed elsewhere [11]. The traditional constraints of reserve, grade blending mill requirements, metal production, processing input capacity and mining capacity are also included in the MIP model.

6. APPLICATION OF THE ALTERNATIVE MIP FORMULATION

The new MIP model is applied to the deposit using, as input, the expected values from 30 jointly simulated orebody models of the deposit, as also used in Section 4. The probabilities are calculated as discussed in the previous section. Figure 6a shows the direct MIP output and Figure 6b shows the output after smoothing the schedule.

The blocks that have 0% and 100% probability of being scheduled in a period are not considered in the MIP model as variables. The values of the blocks that have 100% probability of being scheduled in a period are considered in the model constraints. There are 3001 constraints and 7128 variables, including 2752 binaries in the model considered for the Ni laterite deposit.

Figure 6b shows that the smoothed image of the schedule obtained from the new MIP formulations is a practical pattern. This schedule requires significantly less equipment movement compared with the schedules obtained from the traditional

![Diagram](image)

Fig. 6. Plan view of the schedule obtained by using the proposed new MIP model: (a) is the output of MIP model; (b) is the smoothed image of (a).
Table 4. Summary results from the MIP schedule of the alternative model. T. tons is the total tonnage; O. tons is the ore tonnage; UEV is the undiscounted economic value; S./Ave is the sum of the columns for tonnage and economic values, and the average of the columns representing grades.

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Table 5. Summary results after smoothing the MIP schedule of the alternative model. T. tons is the total tonnage; O. tons is the ore tonnage; UEV is the undiscounted economic value; S./Ave is the sum of the columns for tonnage and economic values, and the average of the columns representing grades.

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model, and the blocks have sufficient space for equipment access. Tables 4 and 5 summarize the schedule as direct output of the MIP and smoothed output, respectively. The tables show that the ore production and grade constraints given in Table 1 are not violated after smoothing the schedule. By directly taking smooth mining patterns into account, the new MIP model is able to keep the number of blocks being rescheduled below the level that would violate model constraints.

7. CONCLUSIONS

An alternative MIP formulation for long-term mine scheduling of multi-element deposits has been presented. The new method is general and may be applied to any open pit mine. For comparing methods in this study, a traditional MIP approach was formulated first, and applied to a Ni-Co laterite deposit using a traditional estimated orebody model. The application shows that a traditional MIP formulation cannot guarantee neither a feasible solution for models of a complex orebody, nor a practical mining schedule. These limitations stem from the failure of the traditional MIP/LP-type
optimization models to take into account the smoothness required in scheduling patterns. The schedule obtained from the traditional MIP model is generally too widely spread over the mineralized deposit to be practical. In the next step of the study, an MIP model was applied to simulated orebody models that reproduce local ore variability. The application shows that optimality is still elusive, as the generated scheduling patterns are impractical and not unique. Furthermore, the application shows that traditional mathematical models in mine optimization do not account for orebody uncertainty, and so lead to different production schedules.

The alternative MIP model proposed in this paper generates feasible scheduling patterns in terms of practical excavation of the blocks during the periods they are scheduled. The excavation requires significantly less movement of equipment compared with the schedules produced by the traditional MIP formulation. The resultant schedule is also feasible in terms of meeting the production targets, such as ore and metal quantities and ore quality parameters. Since the probability of the blocks being scheduled in a period is considered in the optimization, the new schedule is less risky than the traditional in terms of meeting production targets, when considering orebody uncertainty.

REFERENCES